

# Finite-Resistivity Effects on Rayleigh-Taylor Instability of a Stratified Plasma Including Suspended Particles

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The Rayleigh-Taylor (RT) instability of a stratified and viscid magnetoplasma including the effects of finite-resistivity and suspended particles is investigated using normal mode analysis. The horizontal magnetic field and the viscosity of the medium are assumed to be variable. The dispersion relation, which is obtained for the general case on employing boundary conditions appropriate to the case of two free boundaries, is then specialized for the longitudinal and transverse modes. It is found that the criterion of stable stratification remains essentially unchanged and that the unstable stratification for the longitudinal mode can be stabilized for a certain wave number band, whereas the transverse mode remains unstable or all wave numbers which can be stabilized by a suitable choice of the magnetic field for vanishing resistivity. Thus, resistivity is found to have a destabilizing influence on the RT configuration. The growth rates of the unstable RT modes with the kinematic viscosity and the relaxation frequency parameter of the suspended particles have been analytically evaluated. Dust (suspended particles) tends to stabilize the configuration when the medium is considered viscid with infinite conductivity. The kinematic viscosity has a stabilizing influence on the ideal plasma modes.

## I. Introduction

Rayleigh-Taylor (RT) instability, i.e. the instability of a heavier fluid supported by a lighter one whereupon the two interchange their positions, bears some relevance to Laser fusion experiments and has been mentioned as one of the simplest non-trivial instabilities of astrophysical importance. When the heated interior gas in a supernova explosion exerts pressure on the cold shell, increasing its outward momentum, this is equivalent to a low density fluid accelerating a heavier one and RT instability must result as discussed by Spitzer [1]. Chandrasekhar [2] has presented a comprehensive survey of the various investigations carried out on the RT instability problem under various assumptions. He has distinguished two special cases; the instability of two superposed fluids of different densities and the instability of a single fluid having a continuous density stratification, which case he considers equally interesting.

It has been demonstrated by Furth et al. [3], Jukes [4], and many others that the inclusion of finite

resistivity modifies the RT problem and makes possible new unstable modes. Zadoff and Begun [5] have treated the case of two incompressible homogeneous fluids separated by a horizontal boundary in the presence of a uniform horizontal magnetic field. They have discussed the effects of finite resistivity and viscosity of the medium on the growth rate of RT modes and have shown that a finite resistivity does not affect the growth of unstable modes when the wave vector is perpendicular to the magnetic field but that it does increase the growth rate when the wave vector is parallel to the magnetic field. Sundram [6] has considered gravitational instability of a fluid finite resistivity and concluded that the density stratification is unstable for all wave numbers. Bhatia [7] has investigated the case of two superposed fluids of variable viscosity immersed in a horizontal magnetic field, but he has assumed the fluids to be infinitely conducting.

Saffman [8] has initiated the study of dusty gases in magnetohydrodynamics. Alfven and Carlqvist [9] have emphasized the importance of dust in the analysis of the problem of star formation out of interstellar clouds. Michael [10] has pointed out that the presence of dust (suspended particles) tends to

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stabilize the system by reducing the growth rates of the disturbance in context of the Kelvin-Helmholtz instability. Scanlon and Segel [11] have made a detailed study of the hydromagnetic instability in connection with the Benard convection and have examined the influence of suspended particles. Chhajlani et al. [12] have treated the problem of magnetogravitational instability of a finitely conducting plasma in the presence of suspended particles. Sharma and Thakur [13] have considered the RT instability of a stratified partially ionized plasma.

Recently, Chhajlani et al. [14] have incorporated the effect of suspended particles in the RT instability problem of a viscid plasma having an exponentially varying density distribution in the presence of a variable horizontal magnetic field, considering the medium to be infinitely conducting; the growth rate of the unstable Rayleigh-Taylor modes has been evaluated analytically in order to examine the influence of the suspended particle and the viscosity. The approximation of an ideally conducting plasma is valid for an astrophysical plasma but it is often a poor approximation for laboratory plasmas, and hence finite resistivity needs to be incorporated in a more realistic approach. Thus, the simultaneous inclusion of finite resistivity and suspended particles in the analysis of the RT instability of a stratified fluid is certainly interesting, and this has been done in the present study.

## II. Equations of the Problem and Linearization

Consider a homogeneous medium of a gas with suspended particles which is infinite along  $x$ - and  $y$ -directions and bounded by two free surfaces at  $z = 0$  and  $z = d$ . Let  $\mathbf{u}$ ,  $\mathbf{v}$ ,  $N$  and  $\varrho$  denote the gas velocity, the velocity of the particles, the number density of the particles and the gas density, respectively. The particles are assumed to be nonconducting and the gas is considered finitely conducting and viscid. If we assume uniform particle size, spherical shape and small relative velocities between the two phases, then the net effect of the particles on the gas is equivalent to an external force term per unit volume  $KN(\mathbf{v} - \mathbf{u})$ , where  $K = 6\pi\varrho\nu r$  (Stokes' drag formula) and  $r$  and  $\nu$  denote the particle radius and the kinematic viscosity of the gas, respectively. Hence, the relevant equation of

motion for the gas-particle medium is

$$\varrho \frac{d\mathbf{u}}{dt} = -\nabla p + \varrho \nu \nabla^2 \mathbf{u} + KN(\mathbf{v} - \mathbf{u}) + \frac{\mu_e}{4\pi} (\nabla \times \mathbf{H}) \times \mathbf{H} + \mathbf{g} \varrho + \left( \frac{\partial w}{\partial x} + \frac{\partial \mathbf{u}}{\partial z} \right) D\mu, \quad (1)$$

where  $p$ ,  $\mu_e$ ,  $\mathbf{g}(0, 0, -g)$ ,  $\mathbf{H}(H(z), 0, 0)$  denote the pressure, the magnetic permeability of the gas, the gravitational force and the magnetic field, respectively, and  $\mathbf{x} = (x, y, z)$ . The continuity equation of the medium is

$$\frac{\partial \varrho}{\partial t} + (\mathbf{u} \cdot \nabla) \varrho = 0. \quad (2)$$

For an incompressible medium

$$\nabla \cdot \mathbf{u} = 0. \quad (3)$$

The distance between the particles is assumed to be so large that interparticle reactions can be ignored. The buoyancy force on the particles is neglected as its stabilizing influence for two free boundaries is extremely small. The electrical forces on the particles are also neglected. The density changes are small except in the gravity term. Under these approximations, the equations of the motion and continuity for the particles are

$$mN \frac{d\mathbf{v}}{dt} = KN(\mathbf{v} - \mathbf{u}), \quad (4)$$

$$\frac{\partial N}{\partial t} + \nabla \cdot (N\mathbf{v}) = 0, \quad (5)$$

where  $m$  is the mass of the particle.

Finally, the Maxwell's equations for a conducting medium with finite resistivity  $\eta$  are

$$\nabla \cdot \mathbf{H} = 0, \quad (6)$$

$$\frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{H}) + \eta \nabla^2 \mathbf{H}, \quad (7)$$

where  $\eta = c^2/4\pi\sigma$ ;  $\sigma$  is the conductivity and  $c$  the velocity of light.

Now let the initial state of the system be denoted by the subscript "0" and be a quiescent layer with uniform particle distribution. Thus, we have

$$\mathbf{u}_0 = 0, \quad \mathbf{v}_0 = 0, \quad N_0 = \text{constant}. \quad (8)$$

The stability of the initial state is studied by writing solutions to the full equations as the initial

state plus a perturbation term (denoted by primes):

$$\mathbf{u} = \mathbf{u}_0 + \mathbf{u}', \quad \mathbf{v} = \mathbf{v}_0 + \mathbf{v}', \quad N = N_0 + N', \\ \mathbf{H} = \mathbf{H}_0 + \mathbf{h}', \quad p = p_0 + \delta p \quad \text{and} \quad \varrho = \varrho_0 + \delta \varrho, \quad (9)$$

where  $\delta p$  and  $\delta \varrho$  denote perturbations in pressure  $p$  and density  $\varrho$ , respectively. Next, we put the perturbation assumed in (9) in (1)–(7) and linearize them by neglecting products of the perturbations. We also drop the subscript “0” from the equilibrium quantities.

### III. Perturbation Equations

We find that the linearized perturbation equations for the gas-particle medium become

$$\varrho \frac{\partial \mathbf{u}}{\partial t} = -\nabla \delta p + \frac{\mu_e}{4\pi} [\nabla \times \mathbf{h}] \times \mathbf{H} + (\nabla \times \mathbf{H}) \times \mathbf{h} \quad (10) \\ + \mu \nabla^2 \mathbf{u} + KN(\mathbf{v} - \mathbf{u}) + \mathbf{g} \delta \varrho + \left( \frac{\partial w}{\partial x} + \frac{\partial \mathbf{u}}{\partial z} \right) \frac{d\mu}{dz},$$

$$\left( \tau \frac{\partial}{\partial t} + 1 \right) \mathbf{v} = \mathbf{u}, \quad (11)$$

$$\frac{\partial \mathbf{h}}{\partial t} = (\mathbf{H} \cdot \nabla) \mathbf{u} - (\mathbf{u} \cdot \nabla) \mathbf{H} + \eta \nabla^2 \mathbf{h}, \quad (12)$$

$$\nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot \mathbf{h} = 0, \quad (13)$$

$$\frac{\partial}{\partial t} \delta \varrho + (\mathbf{u} \cdot \nabla) \varrho = 0, \quad (14)$$

where  $\tau = m/K$  denotes the relaxation time for the suspended particles and  $\mu$  represents the viscosity of the medium.

We assume the perturbations to vary as

$$\exp(ik_x x + ik_y y + \sigma t), \quad (15)$$

where  $\sigma$  is the growth rate of perturbation and  $k_x$ ,  $k_y$  are the wave numbers along the  $x$ - and  $y$ -directions ( $k^2 = k_x^2 + k_y^2$ ).

Substituting  $\mathbf{v}$  from (11) in (10) and then employing (14) and (15), we obtain components of (10) as follows:

$$[\varrho(\tau\sigma + 1) + mN] \sigma u \\ = -(1 + \sigma\tau) ik_x \delta p + \mu(1 + \sigma\tau)(D^2 - k^2)u \\ + (ik_x w + Du)(1 + \sigma\tau) D\mu \\ + \frac{\mu_e}{4\pi} (1 + \sigma\tau) (DH) h_z, \quad (16)$$

$$[\varrho(\tau\sigma + 1) + mN] \sigma v \\ = -(1 + \sigma\tau) ik_y \delta p + \mu(1 + \sigma\tau)(D^2 - k^2)v \\ + (1 + \sigma\tau)(ik_y w + Dv) D\mu \\ + \frac{\mu_e}{4\pi} H(1 + \sigma\tau)(ik_x h_y - ik_y h_x), \quad (17)$$

$$[\varrho(\tau\sigma + 1) + mN] \sigma w \\ = -(1 + \sigma\tau) D\delta p + \frac{\mu_e}{4\pi} H(1 + \sigma\tau)(ik_x h_z - Dh_x) \\ + \mu(1 + \sigma\tau)(D^2 - k^2)w - \frac{\mu_e}{4\pi} (1 + \sigma\tau) (DH) h_x \\ + 2(1 + \sigma\tau) Dw D\mu + \frac{g}{\sigma} (1 + \sigma\tau) (D\varrho) w, \quad (18)$$

where  $D \equiv d/dz$ .

Similarly, using (15) one obtains from (11)–(14)

$$(\sigma - \eta \nabla^2) h_x = ik_x Hu - w DH, \quad (19)$$

$$(\sigma - \eta \nabla^2) h_y = ik_x H v, \quad (20)$$

$$(\sigma - \eta \nabla^2) h_z = ik_x H w, \quad (21)$$

$$\sigma \delta \varrho = -w D\varrho, \quad (22)$$

$$ik_x u + ik_y v + Dw = 0, \quad (23)$$

$$ik_x h_x + ik_y h_y + Dh_z = 0, \quad (24)$$

where  $\nabla^2 = (D^2 - k^2)$ .

It is easy to eliminate  $\delta p$  from (18) with the help of (16) and (17), and finally substitution of  $h_z$  from (21) in the resulting equation yields following differential equation in the perturbed velocity component  $w$ :

$$\sigma [(\sigma\tau + 1) + \alpha_0] [D(\varrho Dw) - k^2 \varrho w] \\ - \mu(\sigma\tau + 1)(D^2 - k^2)^2 w \\ - (\sigma\tau + 1)(D^2 - k^2) D\mu Dw \\ - (\sigma\tau + 1) D^2 \mu (D^2 + k^2) w \\ + \frac{\mu_e}{4\pi} \frac{k_x^2 (\sigma\tau + 1)}{(\sigma - \eta \nabla^2)} \{H^2 (D^2 - k^2) w + (DH^2) Dw\} \\ + \frac{gk^2}{\sigma} (\tau\sigma + 1) D\varrho w = 0, \quad (25)$$

where  $\alpha_0 = mN/\varrho$  denotes the mass concentration of the suspended particles.

#### IV. Stratification and Boundary Conditions

We shall solve (25) for a gas-particle medium with a continuous stratification of density  $\varrho(z)$  given by

$$\varrho(z) = \begin{cases} \varrho_0 \exp(\beta z), & 0 \leq z \leq d, \\ 0 & \text{elsewhere,} \end{cases} \quad (26)$$

where  $\varrho_0$  is the density at the lower boundary,  $\beta$  is a constant and  $d$  is the depth of the medium. We must note here that a similar stratification is assumed for the suspended particles. For the sake of simplicity in the analysis we consider a similar stratification for the viscosity and magnetic field i.e.

$$\begin{aligned} \mu(z) &= \mu_0 \exp(\beta z), \\ H^2(z) &= H_0^2 \exp(\beta z). \end{aligned} \quad (27)$$

It follows from (27) that the kinematic viscosity and the Alfvén velocity  $V_A = (H_0^2 \mu_0 / 4\pi \varrho_0)^{1/2}$  are constant throughout the medium. Following Chandrasekhar [2] and Bhatia [15] the requisite boundary conditions appropriate for a medium with both bounding surface free are

$$w = 0, \quad D^2 w = 0, \quad D^4 w = 0 \quad \text{at } z = 0 \text{ and } z = d. \quad (28)$$

In fact,  $w$  and all its even derivatives should vanish at the boundaries in this case.

#### V. Dispersion Relation and Discussion

We have divided this section into three sub-sections Va), Vb) and Vc). In all cases we have treated the problem under the assumption that both bounding surfaces are free.

It has been pointed out by Spiegel [16] that free boundaries bear relevance for certain stellar atmospheres. Many authors, including Chandrasekhar [2], have adopted free boundaries because they allow for exact solutions of the problem.

##### a) Viscid Finitely Conducting Dusty Plasma

We shall now obtain a dispersion relation from (26) which represents the combined influence of finite conductivity, viscosity and the suspended particles. For this, we first employ (26) and (27) in (25), and then solve the resulting equation neglecting the effect of heterogeneity on the inertia term.

We thus obtain

$$\begin{aligned} (\sigma \nabla^4 - \eta \nabla^6) w - \frac{k_x^2 V_A^2}{v_0} \nabla^2 w \\ - \frac{(\sigma \tau + 1 + \alpha_0) \sigma}{(\sigma \tau + 1) v_0} (\sigma \nabla^2 - \eta \nabla^4) w \\ - \frac{g \beta k^2}{\sigma v_0} (\sigma - \eta \nabla^2) w = 0, \end{aligned} \quad (29)$$

where  $\nabla^2 = (D^2 - k^2)$  and  $v_0 = \mu_0 / \varrho_0$ .

The differential equation (29), which contains only even derivatives of  $w$ , has to be solved consistent with the boundary condition stated in (28) i.e.

$$w = 0, \quad D^2 w = 0 \quad \text{at } z = 0 \text{ and } z = d.$$

We find that  $w$  and all its even derivatives vanish at  $z = 0$  and  $z = d$ . The proper solution of (29) in view of above boundary condition is

$$w = A \sin(m' \pi z / d), \quad (30)$$

where  $m'$  is an integer.

From (29), inserting the values of  $w$  and its derivatives in accordance to (30), we obtain the dispersion relation

$$\begin{aligned} \sigma(\sigma \tau + 1) v_0 (\sigma L^2 + \eta L^3) + k_x^2 V_A^2 \sigma(\sigma \tau + 1) L \\ + \sigma^2 (\sigma \tau + 1 + \alpha_0) (\sigma L + \eta L^2) \\ - (\sigma \tau + 1) g \beta k^2 (\sigma + \eta L) = 0, \end{aligned} \quad (31)$$

where  $L = [(m' \pi / d)^2 + k^2]$ .

It can be easily verified that for an ideal plasma ( $\eta = 0$ ), it reduces to the dispersion relation obtained by Chhajlani et al. [14]. This can be further simplified and cast in the form

$$\begin{aligned} \sigma^4 \tau + \sigma^3 [L \tau (v_0 + \eta) + (1 + \alpha_0)] \\ + \sigma^2 [v_0 L (\eta L \tau + 1) + (1 + \alpha_0) \eta L + \tau \lambda] \\ + \sigma [\lambda - g \beta k^2 \eta \tau] + \eta (v_0 L^2 - g \beta k^2) = 0, \end{aligned} \quad (32)$$

where  $\lambda = (k_x^2 V_A^2 - g \beta k^2 / L)$ .

Here we introduce  $f_s = (1/\tau)$ , which is the relaxation frequency parameter of the suspended particles. Then (32) assumes the form

$$\begin{aligned} \sigma^4 + \sigma^3 [L (v_0 + \eta) + f_s (1 + \alpha_0)] + \sigma^2 [v_0 L (\eta L + f_s) \\ + (1 + \alpha_0) \eta L f_s + \lambda] + \sigma [f_s \lambda - g \beta k^2 \eta] \\ + \eta f_s (v_0 L^2 - g \beta k^2) = 0, \end{aligned} \quad (33)$$

which is a general dispersion relation representing the combined influence of the viscosity, suspended particles and the finite conductivity on the RT instability of a composite gas-particle magnetized medium.

Now, it is elucidating to discuss (33) for the following special cases:

(i) *Longitudinal mode* ( $k_x = k$ ,  $k_y = 0$ ). This special case concerns perturbations parallel to the magnetic field. We find that the dispersion relation (33) in this case becomes

$$\begin{aligned} \sigma^4 + \sigma^3 [L(v_0 + \eta) + f_s(1 + \alpha_0)] + \sigma^2 [v_0 L(\eta L + f_s) \\ + (1 + \alpha_0) \eta L f_s + \lambda'] + \sigma [f_s \lambda' - g \beta k^2 \eta] \\ + \eta f_s (v_0 L^2 - g \beta k^2) = 0, \end{aligned} \quad (34)$$

where  $\lambda' = (k^2 V_A^2 - g \beta k^2 / L)$ .

Now, when  $\beta < 0$  (stable stratification), all the coefficients of (34) are real and positive and hence will not admit any real positive root or complex root with a real positive part, implying thereby stability of the considered medium. We conclude that a stable density stratification will remain stable even for finite resistivity together with suspended particles and non-zero kinematic viscosity. But when  $\beta > 0$ , which is the criterion for unstable density stratification, the configuration will be unstable if

$$v_0 L^2 < g \beta k^2. \quad (35a)$$

But it can be stabilized if

$$v_0 L^2 > g \beta k^2 \quad \text{and} \quad f_s \lambda' > g \beta k^2 \eta \quad (35b)$$

are satisfied simultaneously, where we have assumed  $\lambda' > 0$ . It can be easily seen from (34) that in the infinite conductivity limit the configuration can be stabilized for those wave numbers which satisfy the inequality

$$k^2 > g \beta / V_A^2 - \left( \frac{m' \pi}{d} \right)^2$$

which is the same as was obtained by Talwar [17].

But for the finitely conducting composite medium one has to satisfy (35b), which involves finite resistivity and viscosity. This indicates destabilization for the longitudinal mode. A close examination of the conditions expressed in (35a) and (35b) reveals that the criterion of instability is indepen-

dent of the medium resistivity, but the stability criterion involves finite resistivity.

(ii) *Transverse mode* ( $k_x = 0$ ,  $k_y = k$ ). Here we consider perturbations which are perpendicular to the direction of the magnetic field. The dispersion relation for this particular mode can be worked out from (33) by setting  $k_x = 0$ ,  $k_y = k$ . We get

$$\begin{aligned} \sigma^4 + \sigma^3 [L(v_0 + \eta) + f_s(1 + \alpha_0)] \\ + \sigma^2 [v_0 L(\eta L + f_s) + (1 + \alpha_0) \eta L f_s - g \beta k^2 / L] \\ + \sigma [-f_s g \beta k^2 / L - g \beta k^2 \eta] \\ + \eta f_s (v_0 L^2 - g \beta k^2) = 0. \end{aligned} \quad (36)$$

We find again that for  $\beta < 0$  the above equation does not possess any positive root or complex root with positive real part. Hence the stable density stratification remains stable. For  $\beta > 0$  (unstable stratification) we find that even if the condition  $v_0 L^2 > g \beta k^2$  is satisfied, the medium can not be stabilized since the coefficient of  $\sigma$  will always be negative and as a result (36) may possess a positive root or complex root with real positive part (Hurwitz's criterion). However, the condition of instability is again given by (35a) which makes the absolute term of (36) negative and therefore will necessarily possess one real positive root which destabilizes the system. Hereafter, we will deal with the general dispersion relation (33) since the case of transverse perturbations is relatively unimportant, which is evident from the foregoing discussion. Now, we shall estimate the growth rate of the unstable RT modes.

It is clear that if (35a) is satisfied, the general dispersion relation (33) will possess at least one positive real root which leads to instability. Let us denote this by  $\sigma_0$ . Then  $\sigma_0$  will satisfy the equation

$$\begin{aligned} \sigma_0^4 + \sigma_0^3 [L(v_0 + \eta) + f_s(1 + \alpha_0)] \\ + \sigma_0^2 [v_0 L(\eta L + f_s) + (1 + \alpha_0) \eta L f_s + \lambda] \\ + \sigma_0 [f_s \lambda - g \beta k^2 \eta] + \eta f_s (v_0 L^2 - g \beta k^2) = 0. \end{aligned} \quad (37)$$

Now to comprehend the implications of finite resistivity, suspended particles and the viscosity of the medium on the growth rate of the unstable Rayleigh-Taylor modes, we evaluate  $d\sigma_0/df_s$  and  $d\sigma_0/dv_0$  from (37) and discuss their nature. We have from (37) assuming  $k$  and  $\alpha_0$  constant

$$\frac{d\sigma_0}{df_s} = \frac{-[\eta L(v_0 L - g \beta k^2) + \sigma_0^3(1 + \alpha_0) + \sigma_0^2\{v_0 L + \eta L(1 + \alpha_0)\} + \sigma_0 \lambda]}{[4\sigma_0^3 + 3\sigma_0^2\{v_0 L + f_s(1 + \alpha_0) + \eta L\} + 2\sigma_0\{v_0 L(\eta L + f_s) + (1 + \alpha_0)f_s \eta L + (f_s \lambda - g \beta k^2 \eta)\}]} \quad (38)$$



Now consider the inequalities

$$\sigma_0^3(1 + \alpha_0) + \sigma_0^2[v_0 L + \eta L(1 + \alpha_0)] + \sigma_0 \lambda \geq \eta L(v_0 L - g \beta k^2) \quad (39)$$

and

$$4\sigma_0^3 + 3\sigma_0^2\{v_0 L + f_s(1 + \alpha_0) + \eta L\} + 2\sigma_0\{v_0 L(\eta L + f_s) + (1 + \alpha_0)f_s \eta L + \lambda\} + (f_s \lambda - g \beta k^2 \eta) \geq 0, \quad (40)$$

where  $\lambda$  is assumed to be positive. If either both upper or both lower signs of the inequalities (39) and (40) hold, then  $d\sigma_0/df_s$  is negative. Thus we infer that the growth rate of unstable RT modes decreases with increasing relaxation frequency parameter of the suspended particles when the mentioned restrictions hold. Thus the conditions (39) and (40) define the region where the suspended particles have a stabilizing influence. But if the upper sign of the inequality (39) and the lower sign of (40), or vice versa, hold simultaneously then the growth rate turns out to be positive. This means, under these limitations suspended particles can increase the growth rate of the unstable RT modes. We observe from (39) and (40) that the stabilizing or destabilizing influence of suspended particles is dependent on the finite resistivity of the medium. Also from (37) we have

$$\frac{d\sigma_0}{dv_0} = \frac{-[\sigma_0^3 L + \sigma_0^2 L(\eta L + f_s) + L^2 f_s \eta]}{4\sigma_0^3 + 3\sigma_0^2 a + 2\sigma_0 b + C}, \quad (41)$$

where  $a = [v_0 L + f_s(1 + \alpha_0) + \eta L]$ ,

$$b = [v_0 L(\eta L + f_s) + f_s \eta L(1 + \alpha_0) + \lambda],$$

$$C = (f_s \lambda - g \beta k^2 \eta).$$

The growth rate of the unstable mode is negative if

$$4\sigma_0^3 + 3\sigma_0^2 a + 2\sigma_0 b + C > 0, \quad (42a)$$

and it is positive if

$$4\sigma_0^3 + 3\sigma_0^2 a + 2\sigma_0 b + C < 0, \quad (42b)$$

where  $\lambda > 0$ . It is readily seen from (41) that for an infinitely conducting plasma ( $\eta = 0$ ) the growth rate is always negative if

$$\lambda > 0 \quad \text{viz.} \quad (k_x^2 V_A^2 > g \beta k^2 / L),$$

whereas for the finitely conducting medium we find that the growth rate is positive under the condition (42b). In the light of above discussion it should,

however, be observed that the kinematic viscosity always suppresses the growth rate of the unstable RT mode for an ideal plasma, but for a finitely conducting plasma it enhances the growth rate under the condition (42b). Thus from the above discussion it is of interest to note that the finite resistivity further destabilizes the system by increasing the growth rate of the unstable modes.

#### b) Dusty Inviscid Finitely Conducting Medium

In this section we deduce the dispersion relation corresponding to an inviscid ( $\nu_0 = 0$ ) plasma. We find that the dispersion relation (33) reduces to

$$\sigma^4 + \sigma^3 [f_s(1 + \alpha_0) + \eta L] + \sigma^2 [(1 + \alpha_0)f_s \eta L + \lambda] + \sigma [f_s \lambda - g \beta k^2 \eta] - f_s \eta g \beta k^2 = 0. \quad (43)$$

In the derivation of the above equation the effect of the mass ( $m$ ) and the size ( $r$ ) of the suspended particles has been included in the analysis through  $f_s$ .

For  $\beta < 0$  and  $\lambda > 0$  the above equation will not admit any real positive root or complex root with real positive part, and so the system remains stable.

For  $\beta > 0$ , (43) will possess at least one real positive root which will destabilize the system for all wave numbers. Let  $\sigma_0$  denote the unstable (positive) root of (43), then

$$\sigma_0^4 + \sigma_0^3 [f_s(1 + \alpha_0) + \eta L] + \sigma_0^2 [(1 + \alpha_0)f_s \eta L + \lambda] + \sigma_0 [f_s \lambda - g \beta k^2 \eta] - f_s \eta g \beta k^2 = 0. \quad (44)$$

We also note from (43) that the unstable density stratification ( $\beta > 0$ ) can not be stabilized. We now proceed to find the growth rate of the unstable mode as we have done in the preceding section. We obtain from (44)

$$\frac{d\sigma_0}{df_s} = \frac{-[(1 + \alpha_0 + \eta L)\sigma_0^3 + \sigma_0^2(1 + \alpha_0)\eta L + \sigma_0\lambda - g\beta k^2 \eta]}{[4\sigma_0^3 + 3\sigma_0^2 a' + 2\sigma_0 b' + C']} \quad (45)$$

where  $a' = f_s(1 + \alpha_0) + \eta L$ ,

$$b' = [(1 + \alpha_0)f_s \eta L + \lambda],$$

$$C' = (f_s \lambda - g \beta k^2) \quad \text{and}$$

$$\lambda = (k_x^2 V_A^2 - g \beta k^2 / L).$$

We find that when  $\lambda > 0$ , the denominator of (45) is always positive and hence  $d\sigma_0/df_s$  shall be positive

or negative according to lower or upper sign of the inequality

$$[(1 + \alpha_0) + \eta L] \sigma_0^3 + \sigma_0^2 (1 + \alpha_0) \eta L + \sigma_0 \lambda \geq g \beta k^2 (\eta). \quad (46)$$

Hence we conclude that if  $\lambda > 0$ , then the growth rate of resistive RT modes may be suppressed as well as enhanced with increasing relaxation frequency parameter of the suspended particles. It is noteworthy from (45) that for an infinitely conducting plasma ( $\eta = 0$ ),  $d\sigma_0/df_s$  turns out to be always positive provided  $\lambda > 0$ . This again indicates that the finite resistivity tends to destabilize the system.

### c) Dusty Plasma with Infinite Conductivity

The present section concerns absence of a magnetic field and may be particularly useful for astrophysical plasma where the approximation of infinite conductivity is considered valid. On substituting  $\eta = 0$  and  $V_A = 0$ , we find that the dispersion relation (33) reduces to

$$\sigma^3 + \sigma^2 [v_0 L + f_s (1 + \alpha_0)] + \sigma [v_0 L f_s - g \beta k^2 / L] - f_s g \beta k^2 / L = 0. \quad (47)$$

We find that for  $\beta < 0$  a stable stratification remains stable since then (47) will not allow for any positive root or complex root with positive real part. For  $\beta > 0$  there is at least one positive real root leading to instability of the system. We denote this root by  $\sigma_0$ , which should satisfy

$$\sigma_0^3 + \sigma_0^2 [v_0 L + f_s (1 + \alpha_0)] + \sigma_0 [v_0 L f_s - g \beta k^2 / L] - f_s g \beta k^2 / L = 0. \quad (48)$$

We first calculate  $d\sigma_0/dv_0$  and thereby examine the influence of the viscosity on the growth rate of the unstable perturbations. We obtain from (48)

$$\frac{d\sigma_0/dv_0}{=} = \frac{(\sigma_0^2 L + \sigma_0 L f_s)}{[3\sigma_0^2 + 2\sigma_0 \{v_0 L + f_s (1 + \alpha_0)\} + (v_0 L f_s - g \beta k^2 / L)]} \quad (49)$$

which is positive or negative according to the upper or lower sign of the inequality

$$3\sigma_0^2 + 2\sigma_0 \{v_0 L + f_s (1 + \alpha_0)\} + v_0 L f_s \geq g \beta k^2 / L. \quad (50)$$

We thus infer that the kinematic viscosity can reduce as well as increase the growth rate of the unstable RT perturbations as determined by (50).

In order to determine the role of the suspended particles in an explicit manner, we calculate  $d\sigma_0/df_s$  from (48), which yields

$$\frac{d\sigma_0/df_s}{=} = \frac{-[\sigma_0^2 (1 + \alpha_0) + \sigma_0 v_0 L] + g \beta k^2 / L}{\{3\sigma_0^2 + 2\sigma_0 [v_0 L + f_s (1 + \alpha_0)] + v_0 L f_s - g \beta k^2 / L\}} \quad (51)$$

which is always negative if either

$$g \beta k^2 / L > 3\sigma_0^2 + 2\sigma_0 [v_0 L + f_s (1 + \alpha_0)] + v_0 L f_s \quad (52)$$

or

$$g \beta k^2 / L < \sigma_0^2 (1 + \alpha_0) + \sigma_0 v_0 L \quad (53)$$

is satisfied.

In the derivation of the above conditions we have made use of the fact that  $\alpha_0 = (mN/\rho)$ , which is the mass concentration of the dust, cannot exceed one. Thus we are led to conclude that the presence of dust always reduces the growth rate of unstable perturbations when the resistivity vanishes. A similar consequence of the presence of dust has been reported by Michael [10] in the context of Kelvin-Helmholtz instability.

Thus we have discussed analytically the implications of simultaneous inclusion of the finite resistivity and the suspended particles on the RT instability of a magnetized and viscid medium having a vertical density stratification. We have found that the criterion of stable density stratification ( $\beta < 0$ ) remains unchanged in all the cases considered. The finite resistivity, in general, has a destabilizing influence on the medium. Our analysis shows that the longitudinal perturbations can be stabilized viz. (35a, b), whereas the transverse perturbations always remain unstable for the case  $\beta > 0$ . The relaxation frequency ( $f_s$ ) of the suspended particles can suppress the growth rate in the region (39), (40), and it can also enhance the growth rate under certain restrictions. It is worth pointing out that the finite resistivity plays an important role in determining these regions. The kinematic viscosity always suppresses the growth rate of unstable RT perturbations for an ideal plasma. On the other hand, it enhances the growth rate when the medium is considered finitely conducting. We have also seen that the dust tends to stabilize the dusty infinitely conducting plasma by reducing the growth rate of the unstable modes.

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- [1] L. Spitzer, *Physical Processes in the Interstellar Medium*, John Wiley, New York 1978, p. 262.
- [2] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*, Clarendon Press, Oxford 1961, Chapter III and X.
- [3] H. P. Furth, J. Killen, and M. N. Rosenbluth, *Phys. Fluids* **6**, 453 (1963).
- [4] J. D. Jukes, *J. Fluid Mech.* **16**, 187 (1963).
- [5] L. N. Zadoff and M. Begun, *Phys. Fluids* **11**, 1238 (1968).
- [6] A. K. Sundram, *Phys. Fluids* **11**, 1709 (1968).
- [7] P. K. Bhatia, *Nuovo Cim.* **19B**, 161 (1974).
- [8] P. G. Saffman, *J. Fluid Mech.* **13**, 120 (1962).
- [9] H. Alfvén and P. Carlqvist, *Astrophys. Space Sci.* **55**, 487 (1978).
- [10] D. H. Michael, *Proc. Camb. Phil. Soc.* **61**, 569 (1965).
- [11] J. W. Scanlon and L. A. Segel, *Phys. Fluids* **16**, 1573 (1973).
- [12] R. K. Chhajlani, P. Vashistha, and S. C. Bhand, *Z. Naturforsch.* **33a**, 1469 (1978).
- [13] R. C. Sharma and K. P. Thakur, *Nuovo Cim.* **71B**, 218 (1982).
- [14] R. K. Chhajlani, R. K. Sanghvi, and P. Purohit, *Z. Naturforsch.* **39a**, 939 (1984).
- [15] P. K. Bhatia and J. M. Steiner, *Math. Anal. Appl.* **41**, 27 (1973).
- [16] E. A. Spiegel, *Astrophys. J.* **141**, 1068 (1965).
- [17] S. P. Talwar, *J. Fluid Mech.* **9**, 581 (1960).